

Kinematic Interpretation of Radial Free-Fall via the Temporal Metric Gradient

Bernal Thalman Cordero^{1*}

¹Independent Researcher, Costa Rica.

Corresponding author(s). E-mail(s): Bernal.thalman@gmail.com;

Abstract

The weak-field limit of the Schwarzschild temporal metric component is sufficient to reproduce the Newtonian radial gravitational acceleration for vertical free-fall. We present a kinematic reformulation in which this acceleration is expressed through the spatial gradient of a local effective temporal-flow rate derived from the temporal metric component. In the weak-field regime, the relevant Christoffel connection reduces to the normalized spatial gradient of this flow rate, yielding the standard inverse-square law without invoking spatial curvature as the direct mechanism. The formulation is extended to the interior of an extended mass, where the standard Newtonian interior field is reinterpreted through the corresponding interior temporal-flow profile. Non-radial motion is discussed via the invariant four-velocity norm, and the standard first post-Newtonian perihelion precession is recovered in the appendix as a consistency check. The work does not modify the Einstein field equations; it isolates the temporal sector to provide an alternative kinematic perspective on the origin of radial gravitational acceleration.

Keywords: gravitation, temporal metric gradient, kinematic acceleration, Schwarzschild metric, Christoffel connection, interior gravitational field

1 Introduction

The geometric formulation of General Relativity (GR) interprets gravitation through the curvature of four-dimensional spacetime [1, 3]. In the standard metric formulation with signature $(-, +, +, +)$, free-fall is governed by the geodesic equation derived from

the Einstein field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c_0^4}T_{\mu\nu}. \quad (1)$$

This framework is observationally robust, accounting for gravitational time dilation, light deflection, perihelion precession, and cosmological dynamics.

The present work does not modify the field equations or the metric formalism. Its objective is to isolate the temporal metric component g_{00} and demonstrate that, in the weak-field limit, the radial acceleration of a freely falling test body can be derived kinematically from the spatial gradient of the local temporal progression rate. This perspective has antecedents in the optical analogy of gravity, where the gravitational field is described through an effective refractive index derived from g_{00} [4], and in pedagogical treatments that identify the gradient of proper time as the driver of free-fall [6].

The central observation is that for purely radial motion in a static, spherically symmetric field, the spatial metric contribution enters only at higher perturbative order. The leading-order acceleration is therefore determined entirely by g_{00} and its radial derivative. By defining a local effective temporal-flow speed $c(r) = c_0\sqrt{-g_{00}(r)}$, one obtains a direct kinematic mapping between $\nabla c(r)$, the Christoffel symbol Γ_{00}^r , and the Newtonian gravitational field.

The paper is structured as follows. Section 2 defines the temporal-flow variable and its gradient. Section 3 derives the weak-field identity linking Γ_{00}^r to $dc(r)/dr$ and recovers Newtonian free-fall. Section 4 extends the kinematic constraint to non-radial motion. Section 5 reinterprets the interior Newtonian field. Section 6 establishes the explicit correspondence with the Schwarzschild solution. Section 7 situates the result in relation to prior work. Section 8 discusses scope and limitations. The appendix derives the standard perihelion precession as a consistency check.

2 Kinematic Foundations of Non-Uniform Temporal Flow

In Minkowski spacetime with signature $(-+++)$, the invariant interval is

$$ds^2 = -c_0^2 dt^2 + dx^2 + dy^2 + dz^2, \quad (2)$$

where c_0 is the invariant vacuum speed of light. The presence of a static mass M modifies the local temporal structure. Let t_0 denote coordinate time at spatial infinity. In the weak-field limit, the proper time interval at radius r is

$$d\tau(r) \approx \left(1 - \frac{GM}{c_0^2 r}\right) dt_0. \quad (3)$$

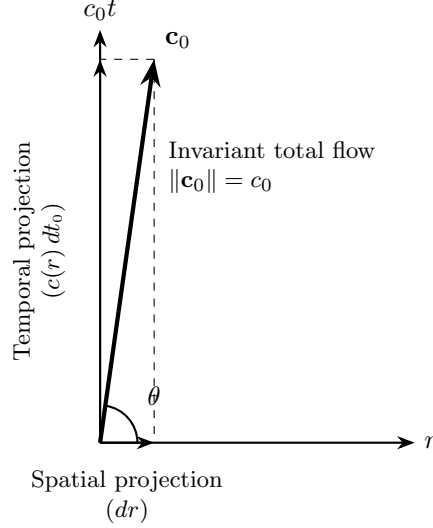


Fig. 1 Heuristic Euclidean decomposition of the invariant spatio-temporal progression. The underlying spacetime geometry is Lorentzian; this diagram is illustrative only.

Multiplying by c_0 defines the local effective temporal-flow speed,

$$c(r) = c_0 \frac{d\tau}{dt_0} = c_0 \left(1 - \frac{GM}{c_0^2 r} \right), \quad (4)$$

satisfying $c(r) < c_0$ for finite r and $c(r) \rightarrow c_0$ as $r \rightarrow \infty$. The radial gradient is

$$\frac{dc(r)}{dr} = \frac{GM}{c_0 r^2}, \quad (5)$$

directed outward. The corresponding vector gradient is

$$\nabla c(r) = \frac{GM}{c_0 r^2} \hat{\mathbf{r}}. \quad (6)$$

2.1 Invariant Norm of the Four-Velocity

For a massive test particle, the invariant norm of the four-velocity is fixed by the Lorentzian metric:

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -c_0^2. \quad (7)$$

This provides the fundamental kinematic constraint connecting temporal and spatial components. In the weak-field static limit, Eq. (7) reduces schematically to a relation between the local temporal projection, governed by $c(r)$, and the spatial displacement. The heuristic decomposition shown in Fig. 1 illustrates this partition; it is a schematic representation and does not imply a Euclidean vector structure on the spacetime manifold.

2.2 Temporal Flow and Its Spatial Gradient

From Eq. (4), the spatial variation of the local temporal-flow speed is

$$\nabla c(r) = \frac{dc(r)}{dr} \hat{\mathbf{r}} = \frac{GM}{c_0 r^2} \hat{\mathbf{r}}. \quad (8)$$

The gravitational acceleration derived below is antiparallel to this gradient:

$$\mathbf{a} \propto -\nabla c(r). \quad (9)$$

Thus free-fall proceeds toward regions of slower temporal progression. The relevant dynamical quantity is the spatial variation dc/dr , not any explicit temporal acceleration $\partial c/\partial t$, which vanishes in static fields.

3 Vertical Free-Fall: Radial Acceleration from Temporal Flow

We analyze radial free-fall toward a spherical mass M . In GR, the geodesic equation governs the motion; for a particle initially at rest, the dominant weak-field contribution to radial acceleration originates in the temporal metric component.

3.1 Temporal Flow Speed and the Metric Component g_{00}

With signature $(-+++)$, $g_{00} < 0$. The local temporal-flow speed is defined by

$$c(r) = c_0 \sqrt{-g_{00}(r)}, \quad (10)$$

so that

$$g_{00}(r) = -\left(\frac{c(r)}{c_0}\right)^2. \quad (11)$$

For the exact Schwarzschild metric,

$$g_{00}(r) = -\left(1 - \frac{2GM}{c_0^2 r}\right), \quad (12)$$

giving

$$c(r) = c_0 \sqrt{1 - \frac{2GM}{c_0^2 r}}. \quad (13)$$

In the weak-field limit, Eq. (13) expands to Eq. (4).

3.2 Kinematic Coupling Identity

For a static, spherically symmetric spacetime with $g_{0r} = 0$, the Christoffel symbol simplifies to

$$\Gamma_{00}^r = -\frac{1}{2} g^{rr} \frac{dg_{00}}{dr}. \quad (14)$$

In the weak-field regime, $g^{rr} \approx 1$ to leading order, so

$$\Gamma_{00}^r \approx -\frac{1}{2} \frac{dg_{00}}{dr}. \quad (15)$$

Using Eq. (11),

$$\frac{dg_{00}}{dr} = -\frac{2c(r)}{c_0^2} \frac{dc(r)}{dr}, \quad (16)$$

and substituting into Eq. (15) yields

$$\Gamma_{00}^r = \frac{c(r)}{c_0^2} \frac{dc(r)}{dr}. \quad (17)$$

With $c(r) \approx c_0$ in the weak-field limit, this reduces to

$$\Gamma_{00}^r \approx \frac{1}{c_0} \frac{dc(r)}{dr} \quad (18)$$

which we term the **kinematic coupling identity**. It is valid strictly in the weak-field limit where $c(r) \approx c_0$ and $g^{rr} \approx 1$.

3.3 Newtonian Free-Fall Acceleration

The radial geodesic equation for a particle initially at rest gives

$$\frac{d^2 r}{dt_0^2} = -c_0^2 \Gamma_{00}^r. \quad (19)$$

Using Eq. (18),

$$a_r = -c_0 \frac{dc(r)}{dr}. \quad (20)$$

With Eq. (5),

$$a_r = -\frac{GM}{r^2} \quad (21)$$

and in vector form,

$$\mathbf{g}(r) = -\frac{GM}{r^2} \hat{\mathbf{r}}. \quad (22)$$

The acceleration is directed opposite to the outward temporal-flow gradient, confirming $\mathbf{g} \propto -\nabla c(r)$.

3.4 Physical Interpretation

The derivation yields the following kinematic picture:

1. The invariant four-velocity norm sets the scale c_0 .
2. A gravitating mass establishes a spatially varying temporal progression $c(r)$.
3. The gradient $dc(r)/dr$ maps directly to g_{00} and to Γ_{00}^r .

4. The resulting coordinate acceleration is an inertial response toward slower temporal progression.

This is a reformulation of the standard weak-field result, not an alternative dynamical law.

4 Non-Radial Motion and Orbital Dynamics

For general trajectories, the spatial projection of the four-velocity acquires radial and transverse components. The invariant norm condition,

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -c_0^2, \quad (23)$$

remains the fundamental constraint. In the equatorial plane ($\theta = \pi/2$) of a static spherically symmetric field, this reads

$$-c_0^2 = g_{00}c_0^2 \left(\frac{dt_0}{d\tau} \right)^2 + g_{rr} \left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\phi}{d\tau} \right)^2. \quad (24)$$

Vertical free-fall corresponds to $d\phi/d\tau = 0$. For orbital motion, the transverse term is non-zero and the four-velocity reorients continuously. The radial gradient $dc(r)/dr$ continues to determine the radial component of the inertial response, while angular momentum conservation follows from the axial symmetry of $c(r)$.

The transition from Eq. (24) to the orbital equation for $u = 1/r$, and the connection to the relativistic perihelion advance, is developed in Appendix A.

5 Interior Field and Gravity Increase Inside Mass

For an extended spherical mass distribution, the Newtonian interior field satisfies Gauss's law,

$$\nabla \cdot \mathbf{g} = -4\pi G\rho. \quad (25)$$

For uniform density ρ , this gives $g(r) = (4\pi G\rho/3)r$, increasing linearly from the center to the surface $r = R$.

Within the temporal-flow framework, this interior behavior is described kinematically by the spatial variation of $c(r)$ inside matter. We refer to this monotonic increase as **Gravity Increase Inside Mass (GIIM)**, denoting the growth of gravitational acceleration from the center toward the surface:

$$g(r) \longrightarrow g_{\max} \quad \text{as} \quad r \rightarrow R^-, \quad (26)$$

where R is the surface radius and $g_{\max}$ the surface acceleration.

GIIM is not an additional force. It is the kinematic description, via $\nabla c(r)$, of the standard interior gravitational field established by the mass distribution $\rho(r)$. A quantitative interior temporal-flow profile requires coupling $c(r)$ to the interior metric (Schwarzschild interior or Tolman–Oppenheimer–Volkoff). That derivation is deferred to future work.

6 Correspondence with the Schwarzschild Connection

In Schwarzschild coordinates (t_0, r, θ, ϕ) , the exterior line element is

$$ds^2 = g_{00}c_0^2 dt_0^2 + g_{rr}dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (27)$$

with

$$g_{00}(r) = -\left(1 - \frac{2GM}{c_0^2 r}\right), \quad g_{rr}(r) = \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}. \quad (28)$$

The definition $g_{00} = -(c(r)/c_0)^2$ gives, to first order in $GM/(c_0^2 r)$,

$$-\left(1 - \frac{GM}{c_0^2 r}\right)^2 \approx -\left(1 - \frac{2GM}{c_0^2 r}\right) = g_{00}(r), \quad (29)$$

confirming consistency in the weak-field regime.

The exact Christoffel connection is

$$\Gamma_{00}^r = \frac{GM}{c_0^2 r^2} \left(1 - \frac{2GM}{c_0^2 r}\right)^{-1}, \quad (30)$$

which reduces to

$$\Gamma_{00}^r \approx \frac{GM}{c_0^2 r^2} \quad (31)$$

in the weak-field limit. Since $(1/c_0) dc(r)/dr = GM/(c_0^2 r^2)$ to the same order,

$$\Gamma_{00}^r \approx \frac{1}{c_0} \frac{dc(r)}{dr}. \quad (32)$$

Thus the temporal-flow gradient reproduces the leading-order radial connection responsible for Newtonian free-fall.

7 Relation to Previous Work

The interpretation of gravitational effects through the temporal metric component has a long history. Einstein's 1911 equivalence-principle argument for light deflection employed a variable speed of light $c(r)$ derived from energy conservation in a static gravitational field [2]. The optical analogy of gravity, in which the field is represented by an effective refractive index $n(r) \propto 1/\sqrt{-g_{00}}$, was developed systematically by Plebański [4] and has been applied to gravitational lensing and signal propagation [5]. Rindler [6] emphasized that the gradient of gravitational time dilation provides the kinematic basis for free-fall in pedagogical terms.

The present work differs from these antecedents in its explicit derivation of the weak-field identity $\Gamma_{00}^r \approx (1/c_0) dc(r)/dr$ and in its systematic isolation of the temporal sector for radial motion. It does not invoke an optical medium or modify electrodynamics; it re-expresses the standard GR result in terms of a single scalar function $c(r)$ and its spatial gradient. The GIIM description of the interior field is the temporal-flow counterpart to the standard Newtonian Gauss-law result.

8 Discussion

The principal result is the weak-field identity linking the temporal-flow gradient to the Christoffel connection,

$$\Gamma_{00}^r \approx \frac{1}{c_0} \frac{dc(r)}{dr}, \quad (33)$$

which connects three descriptions of radial free-fall: the spatial gradient of local temporal progression, the metric component g_{00} , and the affine connection.

The formulation is restricted to the weak-field, radial regime. For non-radial trajectories, strong fields, orbital dynamics, and light propagation, the full metric structure is required; the appendix demonstrates that the standard perihelion precession follows from the complete Schwarzschild geometry, not from g_{00} alone. The temporal-flow interpretation is therefore complementary to, not competitive with, the full geometric formulation.

A complete quantitative theory of the interior temporal-flow profile remains to be developed. Such a theory would require solving the Einstein equations inside matter and extracting $c(r)$ consistently from the interior g_{00} .

9 Conclusions

1. The invariant four-velocity norm establishes the kinematic scale c_0 .
2. The local temporal-flow speed $c(r)$ maps directly to the temporal metric component g_{00} .
3. The spatial gradient $dc(r)/dr$ is related to Γ_{00}^r in the weak-field limit via the kinematic coupling identity.
4. The resulting acceleration $a_r = -GM/r^2$ reproduces the Newtonian field.
5. In vertical free-fall, motion proceeds along $-\nabla c(r)$, toward slower temporal progression.
6. The interior field (GIIM) is described kinematically as the temporal-flow counterpart to the standard Newtonian interior solution.

The central conclusion is that the Newtonian limit of radial gravitational acceleration can be derived kinematically from the spatial gradient of the temporal metric component, offering an alternative perspective on a standard result of General Relativity.

Appendix A Orbital Dynamics and the Perihelion Precession of Mercury

The temporal-flow formulation of the main text isolates the weak-field radial acceleration. The present appendix provides the standard relativistic extension to orbital motion.

A.1 Weak-Field Temporal Flow and the Newtonian Potential

In the weak-field regime, the local temporal-flow rate is

$$c(r) = c_0 \left(1 - \frac{GM}{c_0^2 r} \right), \quad (\text{A1})$$

with corresponding temporal metric component

$$g_{00}(r) = - \left(\frac{c(r)}{c_0} \right)^2 \approx - \left(1 - \frac{2GM}{c_0^2 r} \right). \quad (\text{A2})$$

The Newtonian potential $\Phi(r) = -GM/r$ is recovered through

$$-\frac{d\Phi}{dr} = -\frac{GM}{r^2}, \quad (\text{A3})$$

which matches Eq. (21).

A.2 Relativistic Orbital Equation

Retaining the full Schwarzschild line element,

$$ds^2 = - \left(1 - \frac{2GM}{c_0^2 r} \right) c_0^2 dt_0^2 + \left(1 - \frac{2GM}{c_0^2 r} \right)^{-1} dr^2 + r^2 d\Omega^2, \quad (\text{A4})$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, the geodesic equation in the equatorial plane ($\theta = \pi/2$) with $u = 1/r$ becomes

$$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM}{c_0^2} u^2, \quad (\text{A5})$$

with h the specific angular momentum. The first term yields the Keplerian orbit; the second is the leading post-Newtonian correction.

A.3 Perturbative Solution

The Newtonian solution $u_0(\phi) = (GM/h^2)(1 + e \cos \phi)$, when substituted into the relativistic correction, produces a resonant perturbation. The resulting orbit acquires a shifted phase,

$$u(\phi) \approx \frac{GM}{h^2} [1 + e \cos((1 - \delta)\phi)], \quad (\text{A6})$$

with

$$\delta = \frac{3GM}{c_0^2 p}, \quad p = \frac{h^2}{GM} = a(1 - e^2). \quad (\text{A7})$$

The perihelion advance per revolution is therefore

$$\Delta\phi = \frac{6\pi GM}{a(1 - e^2)c_0^2}. \quad (\text{A8})$$

A.4 Application to Mercury

For Mercury ($a = 5.7909 \times 10^{10}$ m, $e = 0.2056$, $M_{\odot} = 1.9885 \times 10^{30}$ kg), Eq. (A8) gives $\Delta\phi \approx 0.1035''$ per revolution, or approximately $43''/\text{century}$, matching the observed anomalous advance.

A.5 Relation to the Temporal-Flow Formulation

The temporal-flow gradient provides the Newtonian central acceleration and hence the zeroth-order Keplerian orbit. The perihelion advance arises at first post-Newtonian order and requires the full Schwarzschild geometry. The hierarchy is:

$$\text{Temporal-flow gradient} \longrightarrow \text{Newtonian radial acceleration} \longrightarrow \text{Keplerian orbit}, \quad (\text{A9})$$

while the relativistic extension gives

$$\text{Schwarzschild correction} \longrightarrow \text{Orbital phase shift} \longrightarrow \text{Perihelion precession}. \quad (\text{A10})$$

The temporal-flow interpretation is compatible with standard relativistic orbital dynamics when the complete metric is retained.

A.6 Scope

The derivation is intentionally formulated as an extension of the radial analysis, not as an independent derivation of the full Schwarzschild solution. A future development would require deriving the post-Newtonian orbital correction directly from temporal-flow variables alone.

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